UNIT-2

(Lecture-2)

Design of Infinite Impulse Response Digital Filters: Approximation of Derivatives

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Approximation of Derivatives

In this method an analog filter is converted into a digital filter by approximating the differential equation by an equivalent difference equation. The backward difference formula is substituted for the derivative dy(t)/dt at time t = nT.

Thus,

$$\left.\frac{dy(t)}{dt}\right|_{t=nT} = \frac{y(nT) - y(nT - T)}{T} = \frac{y(n) - y(n-1)}{T} \quad \text{---------(1)}$$

Where T is the sampling interval and y(n) = y(nT). The system function of an analog differentiator with

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Approximation of Derivatives

an output dy/dt is H(s)=s, and the digital system that produces the output [y(n)-y(n-1)/T] has the system function $H(z)=(1-z^{-1})/T$. These two can be compare to get the frequency domain equivalent for the relationship in eq(1) as

$$s = \frac{1 - z^{-1}}{T}$$

The second derivative $d^2 y(t)/dt^2$ is replaced by the second backward difference

$$\left.\frac{d^2 y(t)}{dt^2}\right|_{t=nT} = \frac{d}{dT}\left[\frac{dy(t)}{dt}\right]_{t=nT}$$
$$= \frac{[y(nT) - y(nT-T)]/T - [y(nT-T) - y(nT-2T)]/T}{T}$$

Approximation of Derivatives

$$= \frac{y(n) - 2y(n-1) + y(n-2)}{T^2}$$

The equivalent to above equation in frequency domain is

$$s^2 = \frac{1 - 2z^{-1} + z^{-2}}{T^2} = \left(\frac{1 - z^{-1}}{T}\right)^2$$

The i^{th} derivative of y(t) results in equivalent frequency domain relationship

$$s^i = \left(\frac{1 - z^{-1}}{T}\right)^i$$

As a result the digital filter system function can be obtain by the method of approximation of the derivatives as

$$H(z) = H_a(s)\Big|_{s=(1-z^{-1})/T}$$

Where $H_a(s)$ is the system function of the analog filter characterized by the differential equation.

Approximation of Derivatives

Example: Use the backward difference for the derivative to convert the analog low pass filter with system function

H(s) = 1/(s+2)

Solution:

The mapping formula for the backward difference for the derivative is given by $s = \frac{1 - z^{-1}}{T}$

The system response of the digital filter is

$$H(z) = H(s)\big|_{s = \frac{1 - z^{-1}}{T}} = \frac{1}{\left(\frac{1 - z^{-1}}{T}\right) + 2}$$

$$= \frac{T}{1 - z^{-1} + 2T}$$

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